The fractal Package

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**DFA**

_Detrended fluctuation analysis_

**Description**

Performs a detrended fluctuation analysis (DFA) and estimates the scaling exponent from the results. DFA is used to characterize long memory dependence in stochastic fractal time series.

**Usage**

```r
DFA(x, detrend="poly1", sum.order=0, overlap=0,
    scale.max=trunc(length(x)/2), scale.min=NULL,
    scale.ratio=2, verbose=FALSE)
```
Arguments

x  
a vector containing a uniformly-sampled real-valued time series.
detrend  
a character string denoting the type of detrending to use on each block of the
time series. Supported types are:
"polyK" specifies a polynomial fit where K is an integer denoting the order
of the polynomial. For example, if detrend="poly2", a second order
polynomial of the form $x_t = b_0 + b_1t + b_2t^2$ will be used to fit the data in
each block using least squares. The polynomial order must be positive or
zero.
"bridge" specifies bridge detrending. A line connecting the endpoints of
each block is subtracted.
"none" instructs the function to not detrend the data.
Default: "poly1".
overlap  
the overlap of blocks in partitioning the time data expressed as a fraction in $[0,1)$. A positive overlap will slow down the calculations slightly with the (pos-
sible) effect of generating less biased results. Default: 0.
scale.max  
an integer denoting the maximum block size to use in partitioning the data. De-
default: $\text{trunc}(\text{length}(x)/2)$.
scale.min  
an integer denoting the minimum block size to use in partitioning the data. De-
default: for polynomial detrending the default value is $2*(K+1)$. For all other
detrending techniques, the default value is 4 or $\text{length}(x)/4$, whichever is
smaller.
scale.ratio  
the ratio of successive scales. This argument is used as an input to the logScale
function. Default: 2.
sum.order  
an integer denoting the number of differences or cumulative summations to per-
form on the original data before performing a DFA. Differences are specified by
negative integers and cumulative summations by positive integers. For example,
to perform a second order difference, set sum.order=-2. Default: 0.
verbose  
a logical value. If TRUE, the detrending model and processing progress infor-
mation is displayed. Default: FALSE.

Details

The DFA algorithm is implemented as follows:

1 DFA is useful for characterizing long-memory correlations in stochastic fractal time series, i.e.
sequences whose spectral density function $S(f)$ obeys a power law $S \sim |f|^\alpha$ at low frequen-
cies where $0 \leq f \leq 1/2$ is the normalized frequency variable and $\alpha \leq -1$ is the long memory
(scaling) exponent. If the scaling exponent for an original time series is $\alpha > -1$, then (possi-
bly multiple) cumulative summations of the original time series must be performed to increase
the scaling exponent (each cumulative summation decreases the exponent by 2). For exam-
ple, a (single) cumulative summation must be performed on a white noise realization since its
scaling exponent is zero. We also provide the user with the ability to perform (consecutive)
first order differencing operations on the original time series prior to a DFA. Each differencing
operation raises the scaling exponent by 2. Differencing a series is acceptable prior to DFA as
long as the resulting scaling exponent is less than -1.
The series resulting from stage one is uniformly partitioned into blocks of a specified minimum size (scale.min), and each block is (optionally) detrended. The variance of the detrended sequence in each block is calculated, and the collection of variances is averaged to form the scalar value $F_2^{(scale.min)}$ which summarizes the variability of the sequence at the current scale.

Stage two is repeated using successively larger blocks until the largest scale (scale.max) has been reached.

For long-memory processes, we expect to find a linear relation between $\log F(scale)$ and $\log scale$. The slope of the line which best fits a plot of $\log F(scale)$ versus $\log scale$ is defined as the *scaling exponent*.

Value

an object of class fractalBlock.

References


See Also

logScale, fractalBlock.

Examples

```r
## calculate the scaling exponent for a random walk realization
DFA.walk <- DFA(rnorm(1024), detrend="poly1", sum.order=1)
## print the results
print(DFA.walk)
## plot a summary of the results
eda.plot(DFA.walk)
```
FDSimulate  
Simulation of an FD process with time varying model parameters

Description

Creates a realization of a time-varying fractionally differenced (FD) process with a given vector of FD parameters and corresponding vector of innovations variances.

Usage

FDSimulate(delta, innovations.var=1, method="ce", seed=0)

Arguments

delta  
a vector containing time-varying FD parameters.

innovations.var  
a numeric vector or scalar containing (time-varying) FD innovations variances. If a scalar, the value is replicated appropriately. Otherwise, the length of this input should match the length of the delta vector. Default: 1.

method  
a character string defining the method to use in forming the FD realization. Choices are "ce" (circulent emebdding) and "cholesky". Default: "ce".

seed  
a positive integer representing the initial seed value to use for the random number generator. If seed=0, the current time is used as a means of generating a (unique) seed value. Otherwise, the specified seed value is used. Default: 0.

Value

a vector containing a (time-varying) FD process realization corresponding to the input FD model parameters.

S3 METHODS

plot  plot the output object. Optional arguments include:

simulation  Plot the simulated series. Default: TRUE.

delta  Plot the FD parameter as a function of time. Default: TRUE.

innovations.var  Plot the innovations variance as a function of time. Default: TRUE.

print  print the output object.

References


See Also

FDWhittle, wavFDPBlock, wavFDPTime.

Examples

```r
### create a time-varying FD parameter, linearly
### varying from white to pink noise, then jump
### to a red noise plateau
delta <- c(seq(0, 0.5, by=0.01), rep(1,100))

### set the innovations variance to unity
innovation <- rep(1, length(delta))

### simulate a time-varying FD process
z <- FDSimulate(delta=delta, innovation=innovation)
print(z)
plot(z)
```

---

FDWhittle  
*Estimate the Hurst coefficient by Whittle’s method*

Description

Using an estimate of the spectral density function for an input time series, Whittle’s method fits the parameters of a specified SDF model to the data by optimizing an appropriate functional. In this case, the SDF for a fractionally differenced (FD) process model is used and an estimate of \( \delta \), the FD parameter, is returned.

Usage

```r
FDWhittle(x, method="continuous", dc=FALSE, freq.max=0.5, delta.min=-1, delta.max=2.5, sdf.method="direct", ...)
```

Arguments

- `x`: a vector containing a uniformly-sampled real-valued time series.
- `...`: optional SDF estimation arguments passed directly to the SDF function. See help documentation for the SDF function for more information.
- `dc`: a logical value. If FALSE, the DC component of the SDF (corresponding to the sample mean of the series) is not used in optimizing the Whittle functional. Default: FALSE.
- `delta.max`: the maximum value for the FD parameter to use in the constrained optimization problem. Default: 2.5.
- `delta.min`: the minimum value for the FD parameter to use in the constrained optimization problem. Default: -1.
- `freq.max`: the largest normalized frequency of the SDFs used in the analysis. Default: 0.25.
method

A character string indicating the method to be used in estimating the Hurst coefficient (H). Choices are:

"continuous" Whittle's method using a continuous model approach to form the optimization functional. This functional is subsequently implemented via a discrete form of the SDF for an FD process.

"discrete" Whittle's method using (directly) a discrete form of the SDF for an FD process.

Default: "continuous".

sdf.method

A character string denoting the method to use in estimating the SDF. Choices are "direct", "lag window", "wosa" (Welch's Overlapped Segment Averaging), "multitaper". See help documentation for the SDF function for more information. Default: "direct".

Value

Estimate of the FD parameter of the time series.

References


See Also

hurstSpec, FDSimulate.

Examples

```r
set.seed(100)
w <- cumsum(rnorm(1024))
FDWhittle(w, method="discrete", sdf.method="multitaper")
FDWhittle(w, method="continuous", sdf.method="multitaper")
```

FNN

Estimation of the proper embedding dimension for a single-variable time series

Description

Invokes the method of False Nearest Neighbors (FNN) to estimate the minimal embedding dimension of a multivariate data set.

Usage

```r
FNN(x, dimension=5, tlag=NULL, rtol=10, atol=2, olag=1)
```
Arguments

- **x**: a vector containing a uniformly-sampled real-valued time series.
- **atol**: neighbor tolerance based on attractor size. If the Euclidean distance between two neighbor candidates is $Atol$ times larger the estimated "size" of the attractor, then those neighbors are declared as false neighbors. Default: 2.
- **dimension**: the maximal embedding dimension. Default: 5.
- **olag**: orbital lag. The number of points along the trajectory (orbit) of the current point that must be exceeded in order for another point in the phase space to be considered a neighbor candidate. This argument is used to help attenuate temporal correlation in the the embedding which can lead to spuriously low minimal embedding dimension estimates. The orbital lag must be positive or zero. Default: 0.
- **rtol**: false neighbor Euclidean distance tolerance. If the ratio of the Euclidean distances between neighbor candidates in successive embedding dimensions exceeds $Rtol$, then those neighbors are declared as false neighbors. For example, if $Rtol=5$, neighbor candidates that are separated five times more so than in the previous embedding dimension are declared false neighbors. Default: 10.
- **ttag**: the time delay between coordinates. Default: the decorrelation time of the autocorrelation function.

Value

an object of class FNN.

S3 METHODS

- **plot**: plots a summary of the results. Available options are:
  - **xlab**: a character string defining the x-axis label. Default: "Embedding Dimension".
  - **ylab**: a character string defining the y-axis label. Default: "FNN percentage".
  - ... Additional plot arguments (set internally by the `par` function).
- **print**: prints a summary of the results. Available options are:
  - ... Additional print arguments used by the standard `print` function.

References


See Also

FNS, embedSeries, infoDim, corrDim, timeLag, determinism.
Examples

```r
## perform False Nearest Neighbors tests on
## chaotic beam data for embedding dimensions 1
## through 10, using a time delay embedding
## with a time lag of 10 and an orbital lag of
## 15
x <- FNN(beamchaos, tlag=10, olag=15)

## print the results
print(x)

## plot the results
plot(x)
```

FNS

*Estimation of the proper embedding dimension for a single-variable time series*

Description

Invokes the method of False Nearest Strands (FNS) to estimate the minimal embedding dimension of a multivariate data set.

Usage

```r
FNS(x, dimension=5, tlag=NULL, atol=1,
    image.tol=1, olag=1)
```

Arguments

- `x`: a vector containing a uniformly-sampled real-valued time series.
- `dimension`: the maximal embedding dimension. Default: `5`.
- `image.tol`: an integer defining the so-called iterate tolerance. Nearest neighbor pairs (i,J(i)) are separated in time by a point index span dindex = |i-J(i)|, where J(i) represents the index of the nearest neighbor to point i. If a point near i, say k points away also has a nearest neighbor such that |k - J(k)| = dindex +/- M, where M is the iterate tolerance, then the pair (k, J(k)) is added to the current strand. Typically, M=0 or M=1. If M=0, then the difference in index must be exactly the same for each pair included in the strand. If M=1, the index difference is allowed to be 1 point off from the reference pair. Default: `1`.
- `olag`: orbital lag. The number of points along the trajectory (orbit) of the current point that must be exceeded in order for another point in the phase space to be considered a neighbor candidate. This argument is used to help attenuate temporal correlation in the the embedding which can lead to spuriously low minimal embedding dimension estimates. The orbital lag must be positive or zero. Default: `1`.
The time delay between coordinates. Default: the decorrelation time of the auto-correlation function.

Details

The statistic used for determining a false nearest strand (FNS) is based on a Euclidean tolerance supplied by the user (atol). Let $S(d)$ be the mean Euclidean distance in the projected $(d + 1)$th coordinate between strand pairs found to be nearest neighbors in embedding dimension $d$. If $S(d)/A > \text{atol}$, where $A$ is the estimated attractor size, then the strand is considered to be a false strand. $A$ is typically calculated to be the sample standard deviation of the original time series. The $S(d)$ statistic is a measure of the average additional Euclidean distance we gain by embedding the strand in the next dimension, and is used to assess when this extra distance has grown too large, indicating a false strand.

Value

an single-dimensional matrix containing the FNS percentage as a function of embedding dimension.

References


See Also

FNN, embedSeries, infoDim, corrDim, timeLag, determinism.

Examples

```r
## perform False Nearest Strands tests on chaotic beam data for embedding dimensions 1 through 10, using a time delay embedding with a time lag of 10 and an orbital lag of 15
x <- FNS(beamchaos, dim=10, tlag=10, olag=15)

## print the results
print(x)
```
**HDEst**

*Hurvich-Deo estimate of number of frequencies to use in a periodogram regression*

**Description**

Estimates the number of frequencies to use in a periodogram regression estimate of the Hurst parameter $H$ of a long memory time series. Based on estimated spectrum of time series.

**Usage**

```r
HDEst(NFT, sdf, A=0.3, delta=6/7)
```

**Arguments**

- **NFT**: number of points used in the Fourier transform to generate the spectrum.
- **sdf**: estimate of the spectrum of the time series (power, NOT dB).
- **A**: parameter $A$ of Hurvich-Deo model. Default: 0.3 (recommended).
- **delta**: parameter $\delta$ of Hurvich-Deo model. Default: $6/7$ (recommended).

**Value**

estimated optimum number of frequencies "$m$" to use in a periodogram regression estimate of the Hurst parameter $H$.

**References**


**See Also**

`hurstBlock`.

**Examples**

```r
S <- SDF(beamchaos)
HDEst(NFT=length(S), as.vector(S))
```
KDE

**Nonparametric multidimensional probability density function estimation**

**Description**

Given a training matrix, this function estimates a multidimensional probability density function using the Epanechnikov kernel as a smoother. The density function is estimated at a specified and arbitrary set of points, i.e., at points not necessarily members of the training set.

**Usage**

```r
KDE(x, at=NULL, n.grid=100)
```

**Arguments**

- `x`: a matrix whose columns contain the coordinates for each dimension. Each row represents the location of a single point in a multidimensional embedding.
- `at`: the locations of the points over which the KDE is to be calculated. Default: a multidimensional uniform grid of points spanning the training data space (defined by `x`).
- `n.grid`: the number of divisions per dimension to using in forming the default grid when the `at` input is unspecified. Default: 100.

**Details**

The kernel bandwidth is constant (non-adaptive) and is determined by first computing the minimum variance of all dimensions (columns) of `x`. This minimum variance is then used in Scott’s Rule to compute the final bandwidth.

This function is primarily used for estimating the mutual information of a time series and is included here for illustrative purposes.

**Value**

an object of class `KDE`.

**S3 METHODS**

- `eda.plot`: extended data analysis plot showing the original data along with a perspective and contour plot of the resulting KDE. In the case that the primary input `x` is a single variable (a time series), only the KDE is plotted.
- `plot`: plot the KDE or original (training) data. Options are:
  - `style`: a character string denoting the type of plot to produce. Choices are "original", "perspective", and "contour" for plotting the original training data, a perspective plot of the KDE, or a contour plot of the KDE over the specified dimensions. In the case that the primary input `x` is a single variable (a time series), this parameter is automatically set to unity and a KDE is plotted. Default: "original".
dimensions a two-element integer vector denoting the dimensions/variables/columns to select from the training data and resulting multidimensional KDE for perspective and contour plotting. In the case that the primary input $x$ is a single variable (a time series), this parameter is automatically set to unity and a KDE is plotted. Default: $1:2$ for multivariate training data.

xlab character string defining the x-axis label. Default: dimnames of the specified dimensions of the training data. If missing, "X" is used. For univariate training data, the x-axis label is set to the name of the original time series.

ylab character string defining the y-axis label. Default: dimnames of the specified dimensions of the training data. If missing, "Y" is used. For univariate training data, the y-axis label is set to "KDE".

zlab character string defining the z-axis label for perspective plots. Default: "KDE".

grid a logical flag. If TRUE, a grid is plotted for the "original" style plot. Default: "FALSE".

... Optional arguments to be passed directly to the specified plotting routine.

print a summary of the KDE object is printed. Available options are:

justify text justification ala prettPrintList. Default: "left".

sep header separator ala prettyPrintList. Default: ":".

... Additional print arguments sent directly to the prettyPrintList function.

See Also
timeLag.

Examples

## create a mixture of 2-D Gaussian distributed RVs with different means, standard deviations, point density, and orientation.

n.sample <- c(1000, 500, 300)
ind <- rep(1:3, n.sample)
x <- rmvnorm(sum(n.sample),
  mean = rbind(c(-10,-20), c(10,0), c(0,0))[ ind, ],
  sd = rbind(c(5,3), c(1,3) , c(0.3,1))[ ind, ],
  rho = c(0.5, 1, -0.4)[ind])

## perform the KDE
z <- KDE(x)
print(z)

## plot a summary of the results
eda.plot(z)

## form KDE of beamchaos series
plot(KDE(beamchaos),type="l")
Estimate the Hurst coefficient by rescaled range (R/S) method

Description
The series is partitioned into m groups. The R/S statistic is computed as described in the references, the number of groups is increased, and the calculation is repeated. A log-log plot of R/S versus number of groups is, ideally, linear, with a slope related to H, so H can be determined by linear regression.

Usage
RoverS(x, n.block.min=2, scale.ratio=2, scale.min=8)

Arguments
x
a vector containing a uniformly-sampled real-valued time series.
n.block.min
minimum number of blocks in partitioning the data. Must be at least 2. Default: 2.
scale.min
minimum number of data values allowed in a block. This may be restricted so the statistic evaluated within each group is from a reasonable sample. Default: 8.
scale.ratio
ratio of successive scales to use in partitioning the data. For example, if scale.min=8 and scale.ratio=2, the first scale will be 8, the second scale 16, the third scale 32, and so on. Default: 2.

Value
estimated Hurst parameter H of the time series.

References

See Also
hurstBlock, hurstACVF, dispersion.

Examples
RoverS(ocean)
beamchaos

Chaotic beam data

Description

A flexible thin steel beam was mounted vertically to a electromechanical shaker which provided a transverse sinusoidal excitation. The beam tip was placed near two rare earth magnets so as to provide nonlinear buckling forces. The beam was treated with a viscoelastic strip adhered to one side to provide a little damping. The addition of the damping treatment helps to form a more distinguishable fractal structure in phase space embeddings. A laser vibrometer was used to record the beam tip velocity and the analog signal streamed to a National Instruments data acquisition board. The data was sampled at 1000 Hz. The gain of excitation was adjusted until (seemingly) chaotic motion was observed.

References


See Also

ecgrr, eegduke, lorenz, pd5si.

Examples

plot(beamchaos)

chaoticInvariant

Class for chaotic invariant objects

Description

Class constructor for chaoticInvariant.

S3 METHODS

eda.plot plots an extended data analysis plot, which graphically summarizes the process of obtaining a correlation dimension estimate. A time history, phase plane embedding, correlation summation curves, and the slopes of correlation summation curves as a function of scale are plotted.

plot plots the correlation summation curves on a log-log scale. The following options may be used to adjust the plot components:
type  Character string denoting the type of data to be plotted. The "stat" option plots the correlation summation curves while the "dstat" option plots a 3-point estimate of the derivatives of the correlation summation curves. The "slope" option plots the estimated slope of the correlation summation curves as a function of embedding dimension. Default: "stat".

fit  Logical flag. If TRUE, a regression line is overlaid for each curve. Default: TRUE.

grid  Logical flag. If TRUE, a grid is overlaid on the plot. Default: TRUE.

legend  Logical flag. If TRUE, a legend of the estimated slopes as a function of embedding dimension is displayed. Default: TRUE.

...  Additional plot arguments (set internally by the par function).

print  prints a qualitative summary of the results.

See Also

infoDim, corrDim, lyapunov.

Examples

## create a faux object of class chaoticInvariant
faux.data <- list(matrix(rnorm(1024), ncol=2), matrix(1:512))
chaoticInvariant(faux.data,
    dimension = 1:2,
    n.embed = 10,
    n.reference = 50,
    n.neighbor = 35,
    tlag = 10,
    olag = 15,
    resolution = 2,
    series.name = "my series",
    series = 1:10,
    ylab = "log2(C2)",
    xlab = "log2(scale)",
    metric = Inf,
    invariant = "correlation dimension")

corrDim  Correlation dimension

Description

Estimates the correlation dimension by forming a delay embedding of a time series, calculating correlation summation curves (one per embedding dimension), and subsequently fitting the slopes of these curves on a log-log scale using a robust linear regression model. If the slopes converge at a given embedding dimension $E$, then $E$ is the correct embedding dimension and the (convergent) slope value is an estimate of the correlation dimension for the data.
Usage

corrDim(x, dimension=5,
         tlag=timeLag(x, method="acfdecor"), olag=0, resolution=2)

Arguments

x a vector containing a uniformly-sampled real-valued time series or a matrix containing an embedding with each column representing a different coordinate. If the latter, the dimension input is set to the number of columns and the tlag input is ignored.
dimension the maximal embedding dimension. Default: 5.
olag the number of points along the trajectory of the current point that must be exceeded in order for another point in the phase space to be considered a neighbor candidate. This argument is used to help attenuate temporal correlation in the the embedding which can lead to spuriously low correlation dimension estimates. The orbital lag must be positive or zero. Default: length(x)/10 or 500, whichever is smaller.
resolution an integer representing the spatial resolution factor. A value of P increases the number of effective scales by a factor of P at a cost of raising the $\ell_\infty$ norm to the Pth power. For example, setting the resolution to 2 will double the number of scales while imposing and additional multiplication operation. The resolution must exceed unity. Default: 2.
tlag the time delay between coordinates. Default: timeLag(x, method="acfdecor"), the decorrelation time of the autocorrelation function.

Details

To estimate the correlation dimension, correlation summation curves must be generated and subsequently fit with a robust linear regression model to obtain the slopes of these curves on a log-log plot. The dimension at which these slope estimates (appear to) converge reveals the proper embedding dimension for the data and the slope at this (and higher) embedding dimensions is an estimate of the correlation dimension. The function used to fit the correlation summation curves is lmsreg which fits a robust linear model to the data using the method of least median of squares regression. See the on-line help documentation for help on the lmsreg function: in R, lmsreg is found in the MASS package while in S-PLUS it is indigenous and appears in the splus database.

The correlation summation at scale $\varepsilon$ for a given embedding dimension is defined as

$$ C_2(\varepsilon) = \frac{2}{(N-\gamma)(N-\gamma-1)} \sum_{i=1}^{N} \sum_{j=i+\gamma+1}^{N} \Theta(\varepsilon - ||X_i - X_j||), $$

where $\Theta(\cdot)$ is the Heavyside function

$$ \Theta(x) = \{ 0, \text{if } x \leq 0; 1, \text{otherwise} \} $$

and $X_i$ is the $i$th point of a collection of $N$ points in the phase space. The parameter $\gamma$ is the orbital lag.

The algorithm used to calculate the correlation summation is made computationally efficient by using:
1 The $\ell_\infty$ norm to calculate the distance between neighbors in the phase space as opposed to (say) the $\ell_2$ norm which involves taking computationally intense square root and power of two operations. The $\ell_\infty$ norm of the distance between two points in the phase space is the absolute value of the maximal difference between any of the points’ respective coordinates, i.e. if $\mathbf{X} = [z_1, z_2, z_3]^T$ then $||\mathbf{X}||_\infty \equiv \max_i |z_i|$.

2 Bitwise masking and shift operations to reveal the radix-2 exponent of the $\ell_\infty$ norm. This direct means of obtaining the exponent immediately yields the associated scale of the distance between neighbors in the phase space while avoiding costly log operations. The bitwise mask and shift factors are based on the IEEE standard 754 for binary floating-point arithmetic. Initial tests are performed in the code to verify that the current machine follows this standard.

3 A computationally efficient routine to calculate the resulting value of a float raised to a positive integer power. Specifically, the $\ell_\infty$ norm is raised to an integer power ($p$) to effectively increase the spatial resolution by a factor of $p$.

The correlation summation curves $C_2(E, \varepsilon)$ where $E$ is the embedding dimension and $\varepsilon$ is the scale, the correlation dimension curves $D_2(E, \varepsilon)$ can be calculated by

$$D_2(E, \varepsilon) = \frac{\ln C_2(E, 2\varepsilon) - \ln C_2(E, \varepsilon/2)}{\ln 2\varepsilon - \ln \varepsilon/2} = \frac{1}{2} \log_2 \frac{C_2(E, 2\varepsilon)}{C_2(E, \varepsilon/2)}.$$ 

This formulation is used to help suppress numerical instabilities that are present in other numerical derivative schemes such as a first order difference.

As a caveat to the user, the slope estimates of the correlation summation curves will typically display a fair amount of variability and the range of scales over which the slopes are approximately linear may be small. Inasmuch, the correlation dimension estimate should always be interpreted as a subjective summary statistic, even when the original times series is representative of a truly noise-free chaotic response.

Value

an object of class chaoticInvariant.

S3 METHODS

**eda.plot** plots an extended data analysis plot, which graphically summarizes the process of obtaining a correlation dimension estimate. A time history, phase plane embedding, correlation summation curves, and the slopes of correlation summation curves as a function of scale are plotted.

**plot** plots the correlation summation curves on a log-log scale. The following options may be used to adjust the plot components:

- **type** Character string denoting the type of data to be plotted. The "stat" option plots the correlation summation curves while the "dstat" option plots a 3-point estimate of the derivatives of the correlation summation curves. The "slope" option plots the estimated slope of the correlation summation curves as a function of embedding dimension. Default: "stat".

- **fit** Logical flag. If TRUE, a regression line is overlaid for each curve. Default: TRUE.

- **grid** Logical flag. If TRUE, a grid is overlaid on the plot. Default: TRUE.
determinism

**legend** Logical flag. If TRUE, a legend of the estimated slopes as a function of embedding dimension is displayed. Default: TRUE.

... Additional plot arguments (set internally by the par function).

**print** prints a qualitative summary of the results.

---

**References**


---

**See Also**

`infoDim`, `embedSeries`, `timeLag`, `chaoticInvariant`, `lyapunov`, `poincareMap`, `spaceTime`, `findNeighbors`, `determinism`.

---

**Examples**

```r
## calculate the correlation dimension estimates
## for chaotic beam data using a delay
## embedding for dimensions 1 through 10, a
## orbital lag of 10, and a spatial resolution
## of 4.
beam.d2 <- corrDim(beamchaos, olag=10, dim=10, res=4)

## print a summary of the results
print(beam.d2)

## plot the correlation summation curves
plot(beam.d2, fit=FALSE, legend=FALSE)

## plot an extended data analysis plot
eda.plot(beam.d2)
```

---

**Description**

Infers the existence of deterministic structure in a given time series. If fractal structure exists, this function is useful in helping the user decide whether a deterministic chaotic model or stochastic fractal time series model is appropriate for their data.
determinism(x, dimension=6, tlag=NULL, olag=1, scale.min=NULL, scale.max=NULL, resolution=NULL, method="ce", n.realization=10, attach.summary=TRUE, seed=0)

Arguments

x a numeric vector or matrix containing uniformly-sampled real-valued time series.

attach.summary a logical flag. If TRUE, a summary of the results is calculated and attached to the returned object as an attribute named "summary". The summary statistics are calculated using the summary method. Default: TRUE.

dimension an integer defining the maximum embedding dimension to use in analyzing the data. Default: 6.

method a character string representing the method to be used to generate surrogate data. Choices are:
  "aaft" Theiler's Amplitude Adjusted Fourier Transform.
  "phase" Theiler's phase randomization.
  "ce" Davies and Harte's Circulant Embedding.
  "dh" Davison and Hinkley's phase and amplitude randomization.
Default: "ce".

n.realization an integer denoting the number of surrogate realizations to create and analyze for comparison to the ensemble of E-statistics. Default: 10.

olag the number of points along the trajectory of the current point that must be exceeded in order for another point in the phase space to be considered a neighbor candidate. This argument is used to help attenuate temporal correlation in the embedding which can lead to spuriously low correlation dimension estimates. The orbital lag must be positive or zero. Default: length(x)/10 or 500, whichever is smaller.

resolution a numeric value representing the spacing between scales (Euclidean bin size). Default: diff(range(x))/1000.

scale.max a numeric value defining the maximum scale over which the results should be returned. Default: diff(range(x))*sqrt(dimension).

scale.min a numeric value defining the minimum scale over which the results should be returned. Default: min(diff(sort(x)))/1000.

seed a positive integer representing the initial seed value for generating surrogate realizations of the original input time series. These surrogates are used to collect an ensemble of determinism statistics (see DETAILS section for more information). If the specified seed value is positive, the seeds used for generating the surrogate ensemble will be calculated via set.seed(seed);rsample(.Machine$integer.max, size=n.realization). This argument should only be used (by specifying
a positive seed value) if the user wishes to replicate a particular set of results, such as those illustrated in the casebook examples. If \texttt{seed=0}, then the random seeds will be generated based on the current time. Default: 0 (generate the random seeds based on the current time).

tlag

the time delay between coordinates. Default: the decorrelation time of the autocorrelation function.

Details

This function calculates the so-called delta-epsilon test for detecting deterministic structure in a time series by exploiting (possible) continuity of orbits comprising a phase space topology created by a time-delayed embedding of the original time series. This phase space continuity is non-existent for stochastic white noise processes. The delta-epsilon test works by

1 an ensemble of randomized realizations of the original time series, i.e., surrogate data is created.
2 an appropriate phase space statistic (called the E-statistic) is calculated for both the time-delayed embedding of the original time series and the ensemble of surrogates.
3 a comparison of the E-statistic for the original series and the ensemble of surrogate data is made. If there is a separation of the original E-statistic from that of the ensemble, it implies the existence of deterministic structure in the original time series. Conversely, an overlap of E-statistics implies that the original series cannot be discriminated from the ensemble of randomized surrogates and thus it is inferred that the original series is a realization of a random process.

The discriminating E-statistic is calculated as follows: Define

\[ \delta_{j,k} = |z_j - z_k| \]
\[ \epsilon_{j,k} = |z_{j+\kappa} - z_{k+\kappa}| \]
\[ e(r) \equiv \epsilon_{j,k} \quad \text{for } j, k \text{ s.t. } r \leq \delta_{j,k} < r + \Delta r \]

where \( \delta_{j,k} \) is the Euclidean distance (using an infinity-norm metric) between phase space points \( z_j \) and \( z_k \), and \( \epsilon_{j,k} \) is the corresponding separation distance between the points at a times \( \kappa \) points in the future along their respective orbits. These future points are referred to as images of the original pair. The variable \( \kappa \) is referred to as the orbital lag. The increment \( \Delta r \) is the width of a specified Euclidean bin size. Given \( \Delta r \), the distance \( \delta_{j,k} \) is used solely to identify the proper bin in which to store the image distance \( \epsilon_{j,k} \). The average of each bin forms the \( e(r) \) statistic. Finally, the E-statistic is formed by calculating a cumulative summation over the \( e(r) \) statistic, i.e.,

\[ E(r) \equiv \sum e(r). \]

If there exists a distinct separation of the E-statistics for the original time series and the ensemble of surrogate data, it implies that the signal is deterministic. The orbital lag \( \kappa \) should be chosen large enough to sufficiently decorrelate the points evaluated along a given orbit.

Value

an object of class \texttt{determinism}. 

\texttt{determinism}
S3 METHODS

**eda.plot** plots a barplot of the determinism level (expressed as a percentage on $[0,100]$) based on the fraction of overlap between the E-statistics for the original series and that of the ensemble of surrogates. The amount of non-overlap is calculated relative to both the first quartile and extreme values of the E-statistics for the surrogate ensemble.

**plot** plots the E-statistics at small scales of the original series overlaid with those of the ensemble of surrogates (illustrated using boxplots over a subsampled set of the surrogate E-statistics).

**print** print a summary of the analysis.

**summary** produces a summary of the E-statistics for use in the **print** and **plot** and **eda.plot** methods.

References


See Also

*embedSeries, timeLag, spaceTime, surrogate.*

Examples

```r
## perform a determinism test for the beamchaos series. in order to do so, it is vitally
## important to provide the proper orbital lag, which can be estimated as the lag value
## associated with the first common maxima over all contours in a spaceTime plot.
plot(spaceTime(beamchaos))

## we estimate an appropriate olag of 30, and subsequently perform the deterrminism test
beam.det <- determinism(beamchaos, olag=30)
print(beam.det)
plot(beam.det)
eda.plot(beam.det)

## perform a similar analysis for a Gaussian white noise realization
rnorm.det <- determinism(rnorm(1024), olag=1)
print(rnorm.det)
plot(rnorm.det)
eda.plot(rnorm.det)
```
Dispersion analysis

Description

Dispersion analysis measures the standard deviation of aggregated means of a time series taken over logarithmically distributed scales. Dispersion analysis is designed for the analysis of fractional Gaussian noise and should not be used for analyzing fractional Brownian motion.

Usage

dispersion(x, front=FALSE)

Arguments

x a numeric vector or signalSeries object containing a uniformly sampled real-valued time series.

front a logical value. If TRUE, the aggregation is started from the beginning of the time series so that the first points will be included in the result. Otherwise, the aggregation is shifted to include the end of the series. Default: FALSE.

Value

a list containing the scale and dispersion analysis statistic vectors.

References


See Also

DFA, RoverS.

Examples

set.seed(100)
z <- dispersion(rnorm(1024))
plot(log(z$scale),log(z$sd))
**ecgrr**

*Electrocardiogram R-R Interval Data*

**Description**

These data are from the file `rr1.txt` in the ‘RR interval time series modeling: A challenge from PhysioNet and Computers in Cardiology 2002’ site of PhysioNet, sponsored by NIH’s National Center for Research Resources.

The data are the RR intervals (beat-to-beat intervals measured between successive peaks of the QRS complex) for patients in normal sinus rhythm (record 16265 of the MIT-BIH database).

**See Also**

`beamchaos, eegduke, lorenz, pd5si`.

**Examples**

```r
plot(ecgrr)
```

---

**eegduke**

*Electroencephalogram Recordings of a Seizure*

**Description**

EEG of a patient undergoing ECT therapy for clinical depression at the ECT Lab at Duke. The data are fluctuations in electrical potential at a point on the patient’s scalp during seizure, one of several recorded channels. They are measured in microvolts and represent measurements taken at time intervals of roughly 1/40 of a second.

**See Also**

`beamchaos, ecgrr, lorenz, pd5si`.

**Examples**

```r
plot(eegduke)
```
**embedSeries**  
*Creates a delay embedding of a single variable time series*

**Description**

Given the time series \( X_t \), the embedding dimension \( E \), and the time delay \( \tau \), the embedding coordinates are defined as \( X_t, X_{t-\tau}, \ldots, X_{t-(E-1)\tau} \).

**Usage**

```r
embedSeries(x, dimension=NULL, tlag=NULL, series.name=NULL)
```

**Arguments**

- **x** a vector containing a uniformly-sampled real-valued time series.
- **dimension** the maximal embedding dimension. Default: 2.
- **series.name** a character string defining the name of the input time series. Default: `deparseText(substitute(x))`.
- **tlag** the time delay between coordinates. Default: the first zero crossing of the autocorrelation function of \( x \).

**Value**

an object of class `embedSeries`.

**S3 METHODS**

- **as.matrix** convert embedding into matrix object. Usage: `as.matrix(x)`.
- **eda.plot** creates an extended data analysis plot of the data summarizing many of its statistical features. Usage: `eda.plot(x)`.
- **plot** plots the embedding. For embeddings higher than 3, a spin plot of the data is generated. Use the buttons on the spin control panel to control the display. Available options to the plot function are:
  - **dim** The plot dimension. Must be less than or equal to the maximal embedding dimension (number of columns in the embedding matrix). Default: the maximal embedding dimension.
  - ... Additional plot arguments (set internally by the `par` function).
  Usage: `plot(x)`.
- **print** prints a summary of the embedding. Available options are:
  - ... Additional print arguments used by the standard `print` function.
  Usage: `print(x)`.

**See Also**

`timeLag`, `FNN`, `corrDim`, `determinism`.
Examples

```r
## embed the beamchaos series in 10 dimensions
## using a time lag of 15.
z <- embedSeries(beamchaos, tlag=15, dim=10)

## plot the attractor in the phase space
## Not run: plot(z)

## plot the embedding projected down to two
## dimensions
plot(z, dim=2)
```

findNeighbors  

Nearest neighbor search in a multidimensional space

Description

Finds a user specified number of nearest neighbors of a multivariate space defined by the coordinates of the input matrix. Alternatively, the user can specify a maximum distance over which to search for nearest neighbors.

Usage

```r
findNeighbors(x, n.neighbor=NULL, metric=1, max.distance = 0., olag=0, sort.distances=TRUE)
```

Arguments

- `x`: an embedding matrix. Each column of the matrix respresents a single coordinate of the embedding and each row denotes the coordinates of a single point in the embedding.
- `max.distance`: used an alternative to `n.neighbor`, use this parameter to specify the maximum distance to search relative to the current point in the phase space. The metric for the distance is specified separately by the optional `metric` input argument. This arguments must be positive and will only be used if `n.neighbor` is NULL, equal to zero, or less than zero. Default: 0.0.
- `metric`: the metric used to define the distance between points in the embedding. Choices are limited to 1, 2, or Inf which represent an $L_1$, $L_2$, and $L_\infty$ norm, respectively. Default: 1.
- `n.neighbor`: the number of neighbors to find for each point in the embedding. If not NULL, this argument overrides the `max.distance` argument. Default: 2.
- `olag`: an integer scalar representing the orbital lag, which defines the number of points along a trajectory (both forwards and backwards) that are to be excluded as nearest neighbor candidates. This argument helps to prevent temporally correlated data samples from being considered as neighbors to a given point in the embedding. This sitatuation can arise, for example, when a smooth trajectory has been
highly oversampled in time. An orbital lag of 0 implies that the reference point itself may be considered a neighbor candidate. To exclude self-neighbors, set `olag` greater than zero. Default: 0.

`sort.distances`  
a logical flag. If `TRUE`, the neighbors for a given point are sorted by distance from closest to farthest. Default: `TRUE`.

**Details**

An efficient recursive algorithm is used to find all nearest neighbors. First a quadtree is developed to form a recursive partitioning of the embedding matrix, returning row and column index vectors and a list of medians which may be used to sort the embedding matrix. The quadtree is then traversed as an efficient means to find nearest neighbors.

**Value**

a list containing the indices of the original points (corresponding to rows of the embedding matrix), the indices of the neighbors found, and the distance between them. The distance metric is based on that specified by the optional `metric` input argument.

**References**


**See Also**

FNN, FNS.

**Examples**

```r
## Calculate the 10 nearest neighbors for each point of 3-dimensional delayed coordinate embedding of the beamchaos data. Exclude self-neighbors from the output.
embedding <- embedSeries( beamchaos, dim = 3, tlag = 10 )
nn <- findNeighbors( embedding, n.neighbor=10, olag=1 )

## Using the same data, find only those neighbors within a distance 0.1 of the original points based on an L-infinity metric
nn.dist <- findNeighbors( embedding, max.distance=0.1, metric=Inf, olag=1 )
```
fractalBlock

Class constructor for block-dependent estimators for stochastic fractal time series

Description

Class constructor for fractalBlock.

Usage

fractalBlock(domain, estimator, exponent, exponent.name, scale, stat, stat.name, detrend, overlap, data.name, sum.order, series, logfit, sdf=NULL)

Arguments

domain character string defining the domain in which the calculations took place, e.g., in the time or frequency domain.
estimator character string briefly describing the estimator.
extponent numerical value representing the scaling exponent.
extponent.name character string defining the name of the scaling exponent.
scale numeric vector containing the scales used in the analysis.
stat numeric vector containing the statistic calculated in the analysis.
stat.name character vector describing the name for the calculated statistic.
detrend character string describing any series detrending used as a preprocessive measure. NULL values are allowed and signify no detrending.
overlap numeric value on [0,1] defining the fraction of overlap used in adjacent blocks of data during the aggregation process.
data.name character string defining the name of the input series.
sum.order integer representing the sum order.
series a numeric vector containing the input series.
logfit a linear regression model (such as that output by lm, lmsreg, or ltsreg) containing the regression model of the log(scale) versus log(stat) data.
sdf spectral density function. Default: NULL.

S3 METHODS

eda.plot extended data analyis plot of the data. Available options are:
cex character expansion ala par. Default: 1.
col line color index ala par. Default: 2.
plot plots a summary of the results. Available options are:
Henon

**Description**

Calculates the Henon map states using the specified parameter set. The Henon map is defined as

\[ x_n = a - x_{n-1}^2 + by_{n-1} \]

\[ y_n = x_{n-1} \]

A parameter set of \( a = 1.4 \) and \( b = 0.3 \) is known to produce a deterministic chaotic response.

**Examples**

```r
## construct a fractalBlock object
xvar <- 2^(1:10)
yvar <- 0.3^(1:10)
z <- fractalBlock(domain="time", estimator="estimator", exponent=pi, exponent.name="PI", scale=xvar, stat=yvar, stat.name="My Stat", detrend=NULL, overlap=0.2, data.name="My Series", sum.order=-1, series=rnorm(2^10), logfit=lm(y ~ x, data=data.frame(x=log(xvar), y=log(yvar))))

## print the result
print(z)

## plot the result
plot(z)
```
Usage

henon(start=rnorm(2), a=1.4, b=0.3, n.sample=2000, n.transient=10)

Arguments

start  a two-element vector of numeric values denoting the starting values for the X and Y Henon coordinates, respectively.
a  the a parameter. Default: 1.4.
b  the b parameter. Default: 0.3.
n.sample  an integer denoting the number of iterates to create beyond that specified by n.transient. Default: 2000.
n.transient  an integer denoting the number of transient points. These transients are removed from the output. Default: 10.

Value

a list of vectors named x and y corresponding to the X- and Y states of the Henon map, respectively.

See Also

lorenz, lorenz.ode.

Examples

plot(henon(),pch=".",cex=0.1)

hurstACVF

Estimate the Hurst coefficient by regression of scaled asinh plot of ACVF vs log(lag)

Description

Estimates long memory parameters beta (ACVF decay exponent), alpha (Equivalent PPL model spectral density exponent), and H (Equivalent Hurst parameter) by linear regression of scaled asinh of ACVF versus log(lag) over intermediate lag values.

Usage

hurstACVF(x, Ascale=1000, lag.min=1, lag.max=64)

Arguments

x  a vector containing a uniformly-sampled real-valued time series.
Ascale  scale factor for use in the scaled asinh plot. Default: 1000.
lag.max  maximum lag for use in linear regression. Default: 64.
Details

Evaluates autocovariance function (ACVF) of input time series by call to S-Plus function `acf`. Constructs sequence `asinh(Ascale * ACVF) / asinh(Ascale)` and does linear regression (via S-Plus function "lsfit") of this sequence versus log(lag) from `lag.min` to `lag.max`. Draws a plot of the sequence and the fit line. Recommended usage: look at resulting plot. Is the intermediate range approximately linear? If plot is too flat, decrease Ascale. If it decreases to zero too quickly, increase Ascale. Values of Ascale from 10 to `10^6` have been found useful. If `lag.min` and `lag.max` do not bound the range where the sequence is approximately linear then change them and rerun the function to produce a better fit.

Value

a list with three components:

- `beta`: decay exponent of autocovariance function
- `alpha`: spectral density exponent of equivalent PPL model
- `H`: Hurst exponent for equivalent ACVF decay rate

References

A. G. Gibbs and D. B. Percival, Forthcoming paper on the autocovariance of the PPL (pure power law) model. A section of the paper discusses the usefulness of scaled asinh plots.

See Also

`hurstBlock`.

Examples

`hurstACVF(nile, Ascale=1000000, lag.min=3, lag.max=68)`

**hurstBlock**

Hurst coefficient estimation in the time domain

Description

Function to estimate the Hurst parameter H of a long memory time series by one of several methods as specified in input. These methods all work directly with the sample values of the time series (not the spectrum).

- **aggabs**: The series is partitioned into m groups. Within each group, the first absolute moment about the mean of the entire series is evaluated. A measure of the variability of this statistic between groups is calculated. The number of groups, m, is increased and the process is repeated. The observed variability changes with increasing m in a way related by theory to the Hurst parameter H of the input series. For the methods used here, a log-log plot of variability versus number of groups is, ideally, linear, with a slope related to H, so H can be determined by linear regression.
The series is partitioned into m groups. Within each group, the variance (relative to the mean of the entire series) is evaluated. A measure of the variability of this statistic between groups is calculated. The number of groups, m, is increased and the process is repeated. The observed variability changes with increasing m in a way related by theory to the Hurst parameter H of the input series. For the methods used here, a log-log plot of variability versus number of groups is, ideally, linear, with a slope related to H, so H can be determined by linear regression.

**diffvar** The series is partitioned into m groups. Within each group, the variance, relative to the mean of the entire series, is evaluated. The first difference of the variances is then evaluated. A measure of the variability of this statistic between groups is calculated. The number of groups, m, is increased and the process is repeated. The observed variability changes with increasing m in a way related by theory to the Hurst parameter H of the input series. For the methods used here, a log-log plot of variability versus number of groups is, ideally, linear, with a slope related to H, so H can be determined by linear regression.

**higuchi** The series is assumed to have the character of a noise, not a motion. The series is partitioned into m groups. The cumulative sums of the series are evaluated to convert the series from a noise to a motion. Absolute differences of the cumulative sums between groups are analyzed to estimate the fractal dimension of the path. The number of groups, m, is increased and the process is repeated. The result changes with increasing m in a way related by Higuchi’s theory to the Hurst parameter H of the input series. A log-log plot of the statistic versus number of groups is, ideally, linear, with a slope related to H, so H can be determined by linear regression.

**Usage**

```r
hurstBlock(x, method="aggAbs", scale.min=8, scale.max=NULL,
            scale.ratio=2, weight=function(x) rep(1,length(x)), fit=lm)
```

**Arguments**

- **x** a vector containing a uniformly-sampled real-valued time series.
- **fit** a function representing the linear regression scheme to use in fitting the resulting statistics (on a log-log scale). Supported functions are: `lm`, `lmsreg`, and `ltsreg`. See the on-line help documentation for each of these for more information: in R, these are found in the `MASS` package while in S-PLUS they are indigenous and found in the `splus` database. Default: `lm`.
- **method** a character string indicating the method to be used to estimate the Hurst coefficient (H). Choices are:
  - "aggabs" Absolute Values of the Aggregated Series
  - "aggVar" Aggregated Variance Method
  - "diffvar" Differenced Variance Method
  - "higuchi" Higuchi’s Method
  Default: "aggabs".
- **scale.max** an integer denoting the maximum scale (block size) to use in partitioning the series. Default: `length(x)`.
- **scale.min** an integer denoting the minimum scale (block size) to use in partitioning the series. Default: 8.
scale.ratio  ratio of successive scales to use in partitioning the data. For example, if scale.min=8 and scale.ratio=2, the first scale will be 8, the second scale 16, the third scale 32, and so on. Default: 2.

weight  a function with a single required variable (x) used to weight the resulting statistics (x) for each scale during linear regression. Currently, only supported when fit=lm. Default: function(x) rep(1,length(x)).

Value

an object of class fractalBlock.

References


See Also

fractalBlock, hurstSpec, lm.

Examples

```r
## create test series
set.seed(100)
x <- rnorm(1024)
walk <- cumsum(x)

## calculate the Hurst coefficient of a random walk series using various techniques
methods <- c("aggabs","aggvar","diffvar","higuchi")
z <- lapply(methods, function(method, walk){
  hurstBlock(ifelse1(method="higuchi",diff(walk),walk), method=method)
},walk=walk )
names(z) <- methods

## plot results
old.plt <- splitplot(2,2,1)
for (i in 1:4){
  if (i > 1)
    splitplot(2,2,i)
  plot(z[[i]], key=FALSE)
mtext(paste(attr(z[[i]],"stat.name"), round(as.numeric(z[[i]]),3), sep="", H=""), line=0.5, adj=1)
}
par(old.plt)
```
hurstSpec

Hurst coefficient estimation via spectral regression

Description

Function to estimate the Hurst parameter H of a time series by linear regression of the log(spectrum) versus log(frequency) with frequency points accumulated into boxes of equal width on a logarithmic scale and spectrum values averaged over each box.

standard Given an estimate of the SDF for the input time series, this function estimates the Hurst coefficient of the time series by performing a linear regression of log(SDF) versus log(frequency). The range of frequencies to be included in the regression is specified by the dc and freq.max input arguments.

smoothed Given an estimate of the SDF for the input time series, this function estimates the Hurst coefficient of the time series by performing a linear regression of log(SDF) versus log(frequency). The range of frequencies to be included in the regression is specified by the dc and freq.max input arguments. Frequencies are partitioned into blocks of equal width on a logarithmic scale and the SDF is averaged over each block. The number of blocks is controlled by the n.block argument.

robinson Estimates the Hurst coefficient by Robinson’s SDF integration method. Given an estimate of the SDF for the input time series, this function estimates the Hurst coefficient of a time series by applying Robinson’s integral method (typically) to the low-frequency end of the SDF. Use the freq.max argument to define the low-frequency cutoff.

Usage

```r
hurstSpec(x, method="standard", freq.max=0.25, dc=FALSE, n.block=NULL,
        weight=function(x) rep(1,length(x)), fit=lm, sdf.method="direct", ...)
```

Arguments

- **x** a vector containing a uniformly-sampled real-valued time series.
- **...** optional SDF estimation arguments passed directly to the sdf function. See help documentation for the sdf function for more information.
- **dc** a logical value. If FALSE, the DC component of the spectrum (corresponding to the sample mean of the series) is not used in fitting the resulting statistics to estimate the Hurst coefficient. Default: FALSE.
- **fit** a function representing the linear regression scheme to use in fitting the resulting statistics (on a log-log scale). Supported functions are: lm, lmsreg, and ltsreg. See the on-line help documentation for each of these for more information: in R, these are found in the MASS package while in S-PLUS they are indigenous and found in the splus database. Only used when method="standard" or method="smoothed". Default: lm.
- **freq.max** the largest normalized frequency to include in the regression scheme. Default: 0.25.
method a character string indicating the method to be used in estimating the Hurst coefficient (H). Choices are:
"standard" Regression of SDF estimate.
"smoothed" Regression of block averages of the SDF estimate taken over dyadic partitions in frequency.
"robinson" Robinson’s SDF integration method.
Default: "standard".

n.block an integer denoting the number of logarithmic frequency divisions to use in partitioning the estimated SDF. This input argument is only used if method="smoothed". Default: as.integer(floor(logb(length(x),base=2))), which corresponds to the maximum number of decomposition levels possible for a discrete wavelet transformation of the input time series.

sdf.method a character string denoting the method to use in estimating the SDF. Choices are "direct", "lag window", "wosa" (Welch’s Overlapped Segment Averaging), "multitaper". See help documentation for the sdf function for more information. Default: "direct".

weight a function with a single required variable (x) used to weight the resulting statistics (x) for each scale during linear regression. Currently, only supported when fit=lm and is only used when method="standard" or method="smoothed". Default: function(x) rep(1,length(x)).

Value
an object of class fractalBlock.

References

See Also
hurstBlock, fractalBlock, HDEst, lm.

Examples
### create test series
set.seed(100)
x <- rnorm(1024)
walk <- cumsum(x)
## calculate the Hurst coefficient of a random walk series using various techniques. Use a multitaper SDF.

```r
methods <- c("standard", "smoothed")
z <- lapply(methods, function(method, walk){
    hurstSpec(walk, method=method, sdf.method="multitaper")
}, walk=walk)
names(z) <- methods
```

## plot results

```r
old.plt <- par("plt")
for (i in 1:2){
    splitplot(2,1,i)
    plot(z[[i]])
}
par(plt=old.plt)
```

## Robinson’s method

```r
hurstSpec(walk, method="robinson", sdf.method="multitaper")
```

---

### infoDim

**Information dimension**

**Description**

This function estimates the information dimension by forming a delay embedding of a time series, calculating related statistical curves (one per embedding dimension), and subsequently fitting the slopes of these curves on a log-log scale using a robust linear regression model. If the slopes converge at a given embedding dimension \( E \), then \( E \) is the correct embedding dimension and the (convergent) slope value is an estimate of the information dimension for the data.

**Usage**

```r
infoDim(x, dimension=5, tlag=NULL, olag=0, n.density=100, metric=Inf, max.neighbors=as.integer(min(c(round(length(x) / 3), 100))), n.reference=as.integer(round(length(x) / 20)))
```

**Arguments**

- `x`  
  a vector containing a uniformly-sampled real-valued time series.

- `dimension`  
  the maximal embedding dimension. Default: 5.

- `max.neighbors`  
  let \( p = k/N \) for \( 0 < p \leq 1 \) be the mass density where \( N \) is the number of points in the embedding and \( k \) is the number of neighbors found near an arbitrary reference point in the embedding. The `max.neighbors` parameter defines the maximum value for \( k \), regardless of the required density. In the case where the number of neighbors \( k \) required to meet the density \( p \) exceeds
max.neighbors, then \( k \) is limited to \( \text{max.neighbors} \) and (instead) \( N \) is decreased accordingly to \( N' = \lfloor \text{max.neighbors}/p \rfloor \). It is important to note that only the database of neighbors (formed by an efficient kd-tree algorithm) is reduced to \( N' \) points while all \( N \) points in the embedding are considered as neighbor candidates for any given reference point. The point of all this is to reduce the computational burden. Setting \( \text{max.neighbors} \) to a larger value than the default will increase the computational burden but will lessen the error in estimating the average neighborhood radius of all reference points with a (specified) constant neighborhood density. Default: \( \min(c(\text{round}(\text{length}(x) / 3), 100)) \).

metric

the metric used to define the distance between points in the embedding. Choices are limited to \( 1, 2, \) or \( \infty \) which represent an \( L_1, L_2, \) and \( L_\infty \) norm, respectively. Default: \( \infty \).

n.density

the number of points to create in developing the density vector. For a given reference point in the phase space, the density is defined by the relation \( p = k/N \) where \( k \) is the number of neighbors in the phase space and \( N \) is the total number of points in the embedding. To obtain the information dimension statistics, the density is varied logarithmically from \( 1/N \) to 1.0. Default: 100.

n.reference

the number of reference points to use in forming the information dimension statistic. This argument directly specifies the number of equi-dense neighborhoods to average in forming the average neighborhood radius statistic. As with the \( \text{max.neighbors} \) argument, increasing \( \text{n.reference} \) beyond the default will increase the computational burden at the benefit of obtaining (perhaps) less variable statistics. Default: \( \text{round}(\text{length}(x) / 20) \).

olag

the number of points along the trajectory of the current point that must be exceeded in order for another point in the phase space to be considered a neighbor candidate. This argument is used to help attenuate temporal correlation in the embedding which can lead to spuriously low correlation dimension estimates. The orbital lag must be positive or zero. Default: \( \text{length}(x)/10 \) or 500, whichever is smaller.

tlag

the time delay between coordinates. Default: the decorrelation time of the autocorrelation function.

Details

The information dimension (\( D_1 \)) is one of an infinite number of fractal dimensions of a chaotic system. For generalized fractal dimension estimations, correlation integral moments are determined as an average of the contents of neighborhoods in the phase space of equal radius \( \epsilon \). Using this approach, the information dimension for a given embedding dimension \( E \) is estimated via

\[
D_1(E) = \frac{\langle \ln(p) \rangle}{\ln(\epsilon)}
\]

in the limit as \( \epsilon \) approaches zero, where \( \epsilon \) is the radius of an \( E \)-dimensional hypersphere, \( p \) is the density (also known as the mass fraction), and \( \langle \ln(p) \rangle \) is the average Shannon information needed to specify an arbitrary point in the phase space with accuracy \( \epsilon \).

Alternatively, the neighborhoods can be constructed with variable radii but with constant density. The scaling behavior of the average radii of these neighborhoods as a function of density is then used to estimate the fractal dimensions. In this function, we use this constant density approach to calculate the statistics for estimating the information dimension.
For single variable time series, the phase space is approximated with a delay embedding and $D_1(E)$ is thus estimated over statistics gathered for dimensions $1, \ldots, E$. For chaotic systems, these statistics will ‘saturate’ at a finite embedding dimension, revealing both the (estimated) information dimension and an appropriate embedding dimension for the system. A linear regression scheme should be to estimate the $D_1(E)$ using the statistics returned by this function.

Value

an object of class chaoticInvariant.

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eda.plot plots an extended data analysis plot, which graphically summarizes the process of obtaining a information dimension estimate. A time history, phase plane embedding, information dimension curves, and the slopes of information dimension curves as a function of scale are plotted.

plot plots the information dimension curves on a log-log scale. The following options may be used to adjust the plot components:

- **type** Character string denoting the type of data to be plotted. The "stat" option plots the information dimension curves while the "dstat" option plots a 3-point estimate of the derivatives of the information dimension curves. The "slope" option plots the estimated slope of the information dimension curves as a function of embedding dimension. Default: "stat".

- **fit** Logical flag. If TRUE, a regression line is overlaid for each curve. Default: TRUE.

- **grid** Logical flag. If TRUE, a grid is overlaid on the plot. Default: TRUE.

- **legend** Logical flag. If TRUE, a legend of the estimated slopes as a function of embedding dimension is displayed. Default: TRUE.

- ... Additional plot arguments (set internally by the par function).

print prints a qualitative summary of the results.

References


See Also
corrDim, embedSeries, timeLag, chaoticInvariant, lyapunov, poincareMap, spaceTime, findNeighbors, determinism.

Examples

```r
## calculate the information dimension estimates
## for chaotic beam data using a delay
## embedding for dimensions 1 through 10
beam.d1 <- infoDim(beamchaos, dim=10)
```
## print a summary of the results
print(beam.d1)

## plot the information dimension curves without
## regression lines
plot(beam.d1, fit=FALSE, legend=FALSE)

## plot an extended data analysis plot
eda.plot(beam.d1)

---

**lmACF**

*ACF, PACF, and ACVF for various stochastic fractal time series models*

### Description
Computes the autocovariance, autocorrelation or partial autocorrelation sequences for various stochastic fractal time series models.

### Usage

```
lmACF(x, lag.max=32, type="correlation")
```

### Arguments

- **x**: an object of class "lmModel". Use the lmModel function to create this input.
- **lag.max**: the maximum number of lags at which to compute the autocovariance, the autocorrelation or the partial autocorrelation. Default: 32.
- **type**: a character string defining the output type based on the following options:
  - "covariance" - autocovariance sequence
  - "correlation" - autocorrelation sequence
  - "partial" - partial autocorrelation sequence

Default: "correlation".

### Details
The autocovariance sequence is computed using Equation (2.10) of Beran (1994). The autocorrelation sequence is computed by dividing the autocovariance sequence by the variance of the process (i.e., the value of the autocovariance sequence at lag zero). The partial autocorrelation sequence is computed using the Levinson-Durbin recursions.

### Value
an object of class `signalSeries` containing the result.
References


See Also

*lmModel, lmSDF, lmSimulate, ACVStoPACS.*

Examples

```r
models <- c("ppl","fdp","fgn")
lag <- 100
z <- lapply(models, function(x, models, lag)
  { lmACF(lmModel(x), lag=lag)$data},
  models=models, lag=lag)
names(z) <- paste(upperCase(models), "ACF")
stackPlot(seq(0,lag), z, xlab="lag")
title("Stochastic Fractal Model ACFs")
```

---

**lmConfidence**

Confidence intervals for unknown mean

Description

Estimates confidence intervals for an unknown process mean of a time series well modeled by a stochastic fractal process.

Usage

```
lmConfidence(x, model, conf.level=0.95,
  parm.known=FALSE, n.rep=100000)
```

Arguments

- `x` a vector containing a uniformly-sampled real-valued time series or an object of class `wavTransform`.
- `model` an object of class "lmModel". Use the `lmModel` function to create this input.
- `conf.level` confidence interval probability on the interval (0,1). Default: 0.95.
- `n.rep` number of repetitions in a Monte Carlo study. Default: 100000.
- `parm.known` a logical value. Default: FALSE.

Value

an two-element vector defining the low and high limits of the estimated confidence interval.
References


See Also

lmSimulate.

Examples

```r
model <- lmModel("ppl", alpha=-0.9)
lmConfidence(lmSimulate(model), model)
```

### lmConvert

*Stochastic fractal exponent conversion*

**Description**

Estimates the unknown variance of a stochastic fractal process.

**Usage**

```r
lmConvert(x, to="delta")
```

**Arguments**

- **x**: an object of class "lmModel". Use the lmModel function to create this input.
- **to**: a character string defining the target conversion for the given model exponent. Choices are: "alpha","delta","HG","HB","beta". Default: "delta".

**Value**

numeric value representing the converted exponent.

**See Also**

lmModel, lmConfidence.

**Examples**

```r
model <- lmModel("ppl", alpha=-0.99)
lmConvert(model, to="delta")
```
lmModel

Constructor function for objects of class "lmModel"

Description

Packs the parameters defining a specified stochastic fractal time series model into a list and returns
the result.

Usage

lmModel(model, variance.=1.0, delta=0.45,
alpha=-0.9, HG=0.95, HB=0.95,
innovations.var=NULL, Cs=NULL,
bterms=10, dterms=10, M=100)

Arguments

model a character string defining the model type. Choices are
"ppl" Pure power law (PPL) process. A process \{X_t\} is a PPL process if its
SDF is given by
\[ S_X(f) = C_S |f|^\alpha, \quad |f| \leq 1/2, \]
where \( C_S > 0 \). The innovations variance for this process is given by
\( C_S e^{-\alpha(\log(2)+1)} \) (this is the variance of the best linear predictor of the pro-
cess given its infinite past).
"fdp" Fractionally differenced (FD) process. A process \{X_t\} is a FD process
if its SDF is given by
\[ S_X(f) = \frac{\sigma^2}{4 \sin^2(\pi f)^\delta}, \quad |f| \leq 1/2 \]
where \( \sigma^2 \) is the innovations variance, and \( \delta \) is the FD parameter. Thus,
an FD model is completely defined by the innovations variance and FD
parameter.
"fgn" Fractional Gaussian noise (FGN) process. An FGN process \{X_t\} is a
stationary Gaussian process if its ACVF is given by
\[ s_{X,\tau} = \frac{\sigma^2_X}{2} \left( |\tau + 1|^{2H_G} - 2|\tau|^{2H_G} + |\tau - 1|^{2H_G} \right), \]
where \( \sigma^2_X > 0 \) is the variance of the process, while \( 0 < H_G < 1 \) is the
so-called Hurst coefficient. The coefficient \( H_G \) is sometimes called the
self-similarity parameter for a FGN process and is usually designated in the
literature as simply \( H \).
"dfbm" Discrete Fractional Brownian Motion. i.e., regularly-spaced samples
from a FBM process that is defined over the entire real axis.
Cs

pure power law constant. If supplied, this argument is used to compute variance and innovations.var. If not supplied and innovations.var is supplied, then Cs and variance are determined from the innovations.var. Default: NULL.

HB

the Hurst coefficient for a DFBM process. Default: 0.95.

HG

the Hurst coefficient for an FGN process. Default: 0.95.

M

sets the number of terms used in the Euler-Maclaurin summation for calculating the SDF of an FGN process and DFBM process. The default value should be adequate for all values of the Hurst coefficient. Default: 100.

alpha

power law exponent for a PPL model. Default: -0.9.

bterms

an integer used to control the number of primary terms cumulatively summed in computing an ACVS for a PPL process. Default: 10.

delta

the FD parameter. Default: 0.45.

dterms

an integer used to control the number of secondary terms cumulatively summed in computing an ACVS for a PPL process. Default: 10.

innovations.var

innovations variance for an FD or PPL model. If supplied, this argument is used to compute variance and, for a PPL model, Cs. If not supplied and Cs is supplied for a PPL model, then Cs determines innovations.var. If not supplied and Cs is also not supplied for a PPL model or if not supplied for an FD model, then variance determines innovations.var. Default: NULL.

variance

the process variance with a default of unity. If cs or innovations.var is specified, this parameter is set in agreement with those. If the process is nonstationary but has stationary differences, i.e., incrementally stationary, then the process variance is taken to be the variance of the stationary process that is formed by appropriately differencing the nonstationary process.

Value

an object of class lmModel containing a list of model parameters.

References


See Also

lmACF, lmSDF, lmSimulate, lmConvert, lmConfidence, FDWhittle.
Examples

```r
lmModel("ppl", alpha=-2.0)
lmModel("fdp", delta=0.45, innov=1.3)
lmModel("fgn", HG=0.98)
lmModel("dfbm", HB=0.35)
```

SDF for various stochastic fractal time series models

Description

Compute a discretized version of a single-sided parametric spectral density function (SDF) for various stochastic fractal time series models.

Usage

```r
lmSDF(x, sampling.interval=1, n.freq=NULL, n.sample=NULL, with.Nyquist=NULL)
```

Arguments

- `x`: an object of class "lmModel". Use the `lmModel` function to create this input.
- `n.freq`: the number of frequencies at which the SDF is computed (this argument should not be supplied if `n.sample` is supplied). If `n.sample` is non-NULL supplied but `n.freq` is NULL, the actual grid of frequencies is determined by the argument with.Nyquist. Default: if neither `n.sample` nor `n.freq` is specified, `n.freq` defaults to 32.
- `n.sample`: length of a time series. If non-NULL, the spectral resolution is set to `1/(n.sample * sampling.interval)`. Default: NULL (`n.freq` is used to set the spectral resolution instead).
- `sampling.interval`: the sampling interval for the process. The SDF is computed for frequencies on the interval \([0, \text{Nyquist}]\) where Nyquist is `1/(2*sampling.interval)`. The value of `sampling.interval` must be a positive number. Default: 1.
- `with.Nyquist`: a logical flag. If TRUE, the grid of frequencies over which the SDF is evaluated ranges from `1/2*n.freq*sampling.interval` up to the Nyquist frequency; otherwise, the range is from `1/(2*n.freq + 1)*sampling.interval` to just below the Nyquist frequency. The intent of this argument is to mimic the grid of Fourier frequencies for time series with an even or odd sample size by setting with.Nyquist to, respectively, TRUE or FALSE. This argument is only really intended to be used if `n.sample` is not supplied, but `n.freq` is. Default: TRUE.

Details

The SDF is computed as described in Section 7.6 of Percival and Walden (2000), after a possible change of variable to take into account the sampling interval (the discussion in the reference assumes a unit sampling interval).
lmSimulate

Value

an object of class signalSeries containing the SDF.

References


See Also

lmModel, lmACF, lmSimulate.

Examples

```R
old.plt <- par("plt")
models <- c("ppl","fdp","fgn","dfbm")
for (i in seq(along=models)){
  splitplot(2,2,i)
  plot(lmSDF(lmModel(models[i])),
       reference.grid=FALSE, log.axes="xy")
}
par(plt=old.plt)
```

Description

Stochastic fractal time series simulation

Create a simulation of a stochastic fractal time series according to a specified model.

Usage

```R
lmSimulate(x, sampling.interval=1,
            mean=0, n.sample=128, generate.Sj=FALSE,
            Sj=NULL, rn=NULL)
```

Arguments

- `x` an object of class "lmModel". Use the lmModel function to create this input.
- `Sj` a numeric vector of Davies-Harte frequency domain weights used to create the simulation(s). These weights are calculated if not supplied. Default: NULL (not supplied).
generate.Sj a logical value. If TRUE, the Davies-Harte frequency domain weights (Sj) are returned instead of a simulated series. See the references for details. Default: FALSE.

mean the mean value of of the resulting simulation. Default: 0.0.


rn a vector of random normal deviates used to generate uncorrelated random variables for the Davies-Harte simulator. Default: rnorm(2 * length(Sj) - 2).

sampling.interval the sampling interval for the process. The SDF is computed for frequencies on the interval [0, Nyquist] where Nyquist is 1/(2*sampling.interval). The value of sampling.interval must be a positive number. Default: 1.

Details Simulates a stochastic fractal time series via the Davies-Harte technique, which randomizes spectral weights and inverts the result back to the time domain. See the references for more details.

Value an object of class signalSeries containing the simulated series.


See Also \lmModel, \lmACF, \lmSDF, \lmConfidence, \FDSimulate.

Examples

```r
old.plt <- par("plt")
models <- c("ppl","fdp","fgn","dfbm")
for (i in seq(along=models)){
  splitplot(2,2,i)
  plot(lmSimulate(lmModel(models[i])),
       reference.grid=FALSE)
}
par(plt=old.plt)
```
**localProjection**  
*Time series denoising via a local projection filtering technique*

**Description**

Given a time series, $X[t]$, this function performs one iteration of the local projection filtering algorithm as described in Kantz and Schreiber [1]. This noise reduction algorithm is summarized in the following steps:

1. A time lag embedding of dimension `dimension` is formed using $X[t]$, where `dimension` is typically at least twice the dimension at which the underlying dynamics of $X[t]$ become deterministic. At each point in the embedding a neighborhood is determined by a given radius and a given minimum number of required neighbors.

2. Center-of-mass vectors are computed for each embedding point neighborhood and corresponding covariance matrices are computed with respect to the center-of-maxx vectors.

3. The eigenvectors corresponding to the `noise.dimension` smallest eigenvalues are assumed to form a (local) basis for the noise subspace and the projection of the embedding vector onto these "noise" eigenvectors is subtracted from the original embedding vector.

4. The components of the corrected embedding vectors are averaged to compute the overall correction for each point in the original time series.

**Usage**

```r
localProjection(x, dimension=3, tlag=timeLag(x), n.neighbor=dimension + 1,
               max.distance=2*stdev(x), metric=Inf, noise.dimension=1, corr.curve=TRUE)
```

**Arguments**

- `x`  
  a vector containing a uniformly-sampled real-valued time series.

- `corr.curve`  
  boolean argument. If true, the center-of-mass vectors will be corrected for curvature effects. Default: TRUE.

- `dimension`  
  the dimension of the time lag embedding created from the given time series. This value should be at least twice the dimension in which the underlying dynamics of the time series become deterministic. Default: 3.

- `max.distance`  
  the neighbor search for each embedding point finds all neighbors within `max.distance`. Default: 2*stdev(x).

- `metric`  
  the metric used when searching for neighbors. Supported values are 1 (1-norm), 2 (2-norm), and Inf (infinity norm). Default: Inf.

- `n.neighbor`  
  the minimum number of neighbors acceptable to define a neighborhood for each embedding point. If the neighbor search using input `max.distance` does not produce at least this number of neighbors a new search is performed which finds exactly `n.neighbor` neighbors. Default: dimension + 1.

- `noise.dimension`  
  the assumed dimension of the (local) noise subspace. This should be equal to the embedding dimension, `dimension`, minus the dimension where the underlying dynamics of the time series become deterministic. Default: 1.
the time lag used when creating the time lag embedding. Default: 1.

Value
the resulting denoised time series, a vector the same length as the original time series.

References

See Also
`embedSeries`, `medianFilter`, `timeLag`, `FNN`.

Examples
```r
x <- beamchaos@data
x <- x - mean(x)
sigma <- stdev(x)
xnoise <- x + rnorm(length(x)) * sigma / 3
xclean <- localProjection(xnoise, dimension=7, noise.dimension=5, max.distance=3*sigma, n.neighbor=100)

y <- data.frame(xclean, xnoise, x)
stackPlot(x=positions(beamchaos)[][], y=y, ylab=c("denoised","noisy","original"), ylim=range(y))
```

Description
The Lorenz system is defined by the third order set of ordinary differential equations:
\[
\begin{align*}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= rx - y - xz \\
\dot{z} &= -bz + xy
\end{align*}
\]

if the parameter set is \(\sigma = 10, r = 28, b = 8/3\), then the system response is chaotic. The Lorenz is one the hallmark examples used in illustrating nonlinear deterministic chaotic motion.

See Also
`beamchaos`, `ecgrr`, `eegduke`, `pd5si`.

Examples
```r
plot(lorenz[,1], lorenz[,3], pch=".", col="blue")
```
lorenz.ode  

Lorenz system ODEs

Description

Ordinary differential equations for the Lorenz system. See the help documentation for the lorenz data object for more information. A parameter space defined by \( \sigma=10, r=28, \) and \( b=\frac{8}{3} \) is known to produce chaotic motion.

Usage

```r
lorenz.ode(x, sigma=10, r=28, b=8/3)
```

Arguments

- \( x \): a three-element numeric vector representing the current X, Y, and Z states, respectively, of the Lorenz system.
- \( b \): the b parameter. Default: \( \frac{8}{3} \).
- \( r \): the r parameter. Default: 28.
- \( \text{sigma} \): the sigma parameter. Default: 10.

Value

A vector of three values representing the Lorenz states X, Y, and Z, respectively, evaluated with the specified parameter regime.

See Also

`lorenz`, `henon`.

Examples

```r
lorenz.ode(c(0.3,-0.1,1.0))
```

lyapunov  

Local-Global Lyapunov Spectrum Estimation
Description

Estimates the local Lyapunov exponents over a range of user supplied scales and dimensions. The local Lyapunov spectrum is calculated as follows:

1. A delayed embedding of the input time series is formed.
2. For each global reference point (specified by an integer index in the reference matrix) a local Lyapunov spectrum is calculated, one exponent for each dimension from 1 to `local.dimension` and for each (integer) scale specified by the `scale` vector. As the scales grow larger, the Lyapunov exponent estimates tend toward asymptotic values corresponding to the global Lyapunov exponents. The details of how each local spectrum is estimated is given below.
3. The local spectra are then averaged over each global reference point to stabilize the results.

Each local spectrum is obtained by estimating the eigenvalues of the so-called Oseledec matrix, which is formed through a matrix product of successive local Jacobians with the transpose of the Jacobians. The number of Jacobians in the product is equivalent to the scale. Each Jacobian is formed by fitting a local neighborhood of points (relative to a some reference point) with a multidimensional polynomial of order `polynomial.order`. The number of neighbors found for each reference point in the embedding is chosen to be twice the polynomial order for numerical stability. To further stabilize the results, a local Lyapunov spectrum is formed for each local reference point.

Usage

```r
lyapunov(x, tlag=NULL, dimension=5, local.dimension=3, reference=NULL, n.reference=NULL, olag=2, sampling.interval=NULL, polynomial.order=3, metric=Inf, scale=NULL)
```

Arguments

- `x`: a vector containing a uniformly-sampled real-valued time series.
- `dimension`: an integer representing the embedding dimension. Default: 5.
- `local.dimension`: an integer representing the dimension (number of) local Lyapunov exponents to estimate. This value must be less than or equal to the embedding dimension. Default: 3.
- `metric`: the metric used to define the distance between points in the embedding. Choices are limited to 1, 2, or Inf which represent an $L_1$, $L_2$, and $L_\infty$ norm, respectively. Default: Inf.
- `n.reference`: the number of neighbors to use in in developing the kd-tree (used as a quick means of finding nearest neighbors in the phase space). These neighbors are collected relative to the reference points. This value must be greater than 10. Default: min(as.integer(round(length(x)/20)), 100).
- `olag`: the number of points along the trajectory of the current point that must be exceeded in order for another point in the phase space to be considered a neighbor candidate. This argument is used to help attenuate temporal correlation in the embedding which can lead to spuriously low correlation dimension estimates. The orbital lag must be positive or zero. Default: length(x)/10 or 500, whichever is smaller.
polynomial.order

the order of the polynomial to use in fitting data around reference points in the phase space. This polynomial fit will be used to form the Jacobians which are in turn used to calculate the Lyapunov exponents. Default: 3.

reference

a vector of integers representing the indices of global reference points to use in estimating the local Lyapunov spectrum. A local spectrum is estimated around each global reference point, and all the local spectra are then averaged to stabilize the results. These global reference points should be chosen such that they are far apart in time. Default: Five indices uniformly distributed on the interval [1,M], where M = Ne - max(scale) - n.reference - 2 and Ne is the number of embedding points.

sampling.interval

a numeric value representing the interval between samples in the input time series. Default: deltat(x).

scale

a vector of integers defining the scales over which the local Lyapunov exponents are to be estimated. As this scale increases, one expects the local Lyapunov exponent estimates to converge towards the global estimates. All scales must be greater than one. Default: as.integer(2^{seq(min(floor(logb(scale.max,2)) - 2 , 10)) - 1}) where scale.max = Ne - 2 - n.reference.

tlag

the time delay between coordinates. Default: the decorrelation time of the auto-correlation function.

Value

an object of class FNN.

S3 METHODS

plot plots a summary of the results. Available options are:

... Additional plot arguments (set internally by the par function).

print prints a summary of the results. Available options are:

... Additional print arguments used by the standard print function.

summary summarizes the results.

References


See Also

embedSeries, infoDim, corrDim, timeLag, FNN.
### Examples

```r
## Calculate the local Lyapunov spectrum for the
## beamchaos series
z <- lyapunov(beamchaos)

## print the results
print(z)

## summarize the results
summary(z)

## plot the results
plot(z)
```

---

#### medianFilter

**Median filtering of a time series**

**Description**

Given a filter order \( P \), the output of applying a median filter to a time series \( X[t] \) for \( t = 0, \ldots, N-1 \) is

- **\( P \) odd:** \( Y[k] = \text{median}(X[k - (P - 1)/2, \ldots, k + (P - 1)/2]) \)
- **\( P \) even:** \( Y[k] = \text{median}(X[k - P/2, \ldots, k + P/2 - 1]) \)

for \( k = 0, \ldots, N - 1 \). Thus, median filtering replaces the \( k^{th} \) value of the time series with the median of the time series over an \( P \)-point window centered about point \( k \). In the case where a portion of the window exceeds the boundaries of the time series, the values outside the boundaries are ignored in the median value calculation.

**Usage**

```r
medianFilter(x, order=2)
```

**Arguments**

- **\( x \)**: a vector containing a uniformly-sampled real-valued time series.
- **\( order \)**: the median filter order. This argument defines the size of the windows over which the median values are calculated. The filter order must be positive and less than twice the length of the time series. Default: 2.

**Value**

a vector containing the result and of the same length as the original time series.

**See Also**

`localProjection`.
pd5si

Examples

```r
x <- beamchaos@data
x <- x - mean(x)
sigma <- stdev(x)
xnoise <- x + rnorm(length(x)) * sigma / 3
xclean <- medianFilter(xnoise, order=10)
y <- data.frame(xclean, xnoise, x)

stackPlot(x=positions(beamchaos)[], y=y,
          ylab=c("denoised","noisy","original"),
          ylim=range(y))
```

---

pd5si  

**Gait stride intervals for a patient with Parkinson’s Disease**

Description

These data are from the file `pd5-si.txt` in the ‘gait in aging and disease’ database of PhysioBank, which is a large collection of physiologic signals maintained as a public service at [http://www.physionet.org/physiobank](http://www.physionet.org/physiobank) by NIH’s National Center for Research Resources. The following quote from this Web site describes how the stride intervals were obtained. *Subjects walked continuously on level ground around an obstacle-free path. The stride interval was measured using ultra-thin, force sensitive resistors placed inside the shoe. The analog force signal was sampled at 300 Hz with a 12 bit A/D converter, using an ambulatory, ankle-worn microcomputer that also recorded the data. Subsequently, the time between foot-strikes was automatically computed. The method for determining the stride interval is a modification of a previously validated method that has been shown to agree with force-platform measures, a ‘gold’ standard.*

Data were collected from the healthy subjects as they walked in a roughly circular path for 15 minutes, and from the subjects with Parkinson’s disease as they walked for 6 minutes up and down a long hallway.

See Also

`beamchaos, ecgrr, eegduke, lorenz`.

Examples

```r
plot(pd5si)
```
poincareMap

Create a Poincare map

Description
Create a map using the extrema of a scalar time series.

Usage
poincareMap(x, extrema="min", denoise=FALSE)

Arguments
x
a vector holding a scalar time series.
denoise
a logical value. If TRUE, the data is first denoised via waveshrink prior to analysis. Default: FALSE.
extrema
the type of extrema desired. May be "min" for minima, "max" for maxima, or "all" for both maxima and minima. Default: "min".

Details
This function finds the extrema of a scalar time series to form a map. The time series is assumed to be a uniform sampling of \( s(t) \), where \( s(t) \) is a (possibly noisy) measurement from a deterministic non-linear system. It is known that \( s'(t), s''(t), \ldots \) are legitimate coordinate vectors in the phase space. Hence the hyperplane given by \( s'(t) = 0 \) may be used as a Poincare surface of section. The intersections with this plane are exactly the extrema of the time series. The time series minima (or maxima) are the interesections in a given direction and form a map that may be used to estimate invariants, e.g., correlation dimension and Lyapunov exponents, of the underlying non-linear system.

The algorithm used to create a Poincare map is as follows.

1 The first and second derivatives of the resulting series are approximated via the continuous wavelet transform (CWT) using the first derivative of a Gaussian as a mother wavelet filter (see references for details).
2 The locations of the local extrema are then estimated using the standard first and second derivative tests on the CWT coefficients at a single and appropriate scale (an appropriate scale is one that is large enough to smooth out noisy components but not so large as to the oversmooth the data).
3 The extrema locations are then fit with a quadratic interpolation scheme to estimate the amplitude of the extrema using the original time series.

Value
a list where the first element (location) is a vector containing the temporal locations of the extrema values, with respect to sample numbers 1,\ldots,N, where \( N \) is the length of the original time series. The second element (amplitude) is a vector containing the extrema amplitudes.
References


See Also

`embedSeries`, `corrDim`, `infoDim`.

Examples

```r
## Using the third coordinate (eqn{z} state) of a chaotic Lorenz system, form a discrete map
## using the series maxima. Embed the resulting extrema in a 2-dimensional delay embedding
## (with delay=1 for a map). The resulting plot reveals a tent map structure common to
## Poincare sections of chaotic flows.
z <- poincareMap(lorenz[,3], extrema="max")
z <- embedSeries(z$amplitude, tlag=1, dimension=2)
plot(z, pch=1, cex=1)
```

**spaceTime**

*Space time separation plot*

Description

This function can be used to generate contours of a space time separation plot. This plot type is a visual tool which can help to determine the correlation time for a particular delay embedding of a given time series.

Usage

```r
spaceTime(x, dimension=2, tlag=timeLag(x, method="acfdecor"),
          olag.max=as.integer(min(500,length(x)/20)), probability=0.1)
```

Arguments

- `x` a vector holding a scalar time series
- `dimension` the desired embedding dimension. Default: 2.
- `olag.max` an integer representing the maximum orbital lag to use in forming the results. Default: `as.integer(min(500,length(x)/20))`.
- `probability` a positive numeric scalar on the interval (0,1) which gives the probability associated with the first contour. This input determines the number of contours to be generated (see the output description below). Default: 0.1.
- `tlag` the delay used to create the delay embedding. Default: `timeLag(x, method="acfdecor")`. 
Details
Each contour, $C_p(\Delta t)$, in a space time separation plot corresponds to a particular probability, $p$, and gives spatial distance between pairs of phase space vectors as a function of their temporal separation. In particular, any pair of vectors separated in time by $\Delta t$ are separated in the phase space by distance $C_p(\Delta t)$ with probability $p$.

Value
an object of class `spaceTime`.

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`as.matrix` convert the output to a matrix.

`eda.plot` plot a summary of the space-time contours including a density function estimate of the median contour in addition to a suggested range of suitable orbital lags. In the latter case, the most populous values of the median contour are highlighted by a cross-hatched area that covers a plot of the median curve. The suggested range for a suitable orbital lag is based on the range of values that first escape this cross-hatched region. Optional parameters include:

- `type` An integer denoting the type of line to plot ala the `par` function. Default: "1" (solid line).
- `density` The density of the cross-hatched area ala the `polygon` function. Default: 10.
- `add` A logical flag. If TRUE, the plot is added using the current `par` settings. Otherwise, the `par` settings are adjusted as needed. Default: FALSE.
- `...` Additional parameters sent directly to the `par` function.

`plot` plot the space-time contours for the given `spaceTime` object. Optional arguments include:

- `lty` Line type ala the `par` function. Default: 1.
- `color` A vector of integers defining the contour line colors. Default: 1:8.
- `xlab` A character string denoting the x-axis label. Default: "Orbital Lag".
- `ylab` A character string denoting the y-axis label. Default: "Spatial Separation".
- `main` A character string denoting the title label. Default: NULL (no title).
- `cex` Character expansion value ala `par`. Default: 1.
- `pch` An integer representing the plot character ala `par`. Default: ".".
- `add` A logical flag. If TRUE, the plot is added using the current `par` settings. Otherwise, the `par` settings are adjusted as needed. Default: FALSE.
- `...` Additional parameters sent directly to the `par` function.

`print` print a summary of the `spaceTime` object.

References

See Also
`embedSeries`, `determinism`, `timeLag`.
Examples

```r
## Using the beamchaos data calculate the space
time separation contours for probabilities
## 1/10, 2/10, ..., 1, for a 3-dimensional
delay embedding with delay of 10. Plot the
resulting contours, which will reveal
periodicity in the data. From the top
contour in the plot, which corresponds to
probability 1, we can conclude that any two
vectors in the chosen delay embedding which
are separated in time by approximately 90
time steps are separated by a distance of at
least 8 in the phase space.
z <- spaceTime(beamchaos, dim=3, tlag=10,
  olag.max=500, probability=1/10)

## print the results
print(z)

## plot the results
plot(z)

## extended data analysis plot
eda.plot(z)
```

---

**Description**

The Priestley-Subba Rao (PSR) test for nonstationarity is based upon examining how homogeneous a set of spectral density function (SDF) estimates are across time, across frequency, or both. The original test was formulated in the terms of localized lag window SDF estimators, but such estimators can suffer from bias due to leakage. To circumvent this potential problem, the SDF estimators are averages of multitaper SDF estimates using orthogonal sinusoidal tapers.

**Usage**

```r
stationarity(x, n.taper=5,
  n.block=max(c(2, floor(logb(length(x), base=2)))),
  significance=0.05,
  center=TRUE, recenter=FALSE)
```

**Arguments**

- `x` a vector containing a uniformly-sampled real-valued time series.
- `center` a logical value. If `TRUE`, the mean value of the data is subtracted from the original series prior to analysis. Default: `TRUE`. 
n.block the number of non-overlapping blocks with which the time series will be uniformly divided. If the number of samples in the time series is not evenly divisible by n.block, then those samples at the of the time series that do not fit into the last block are ignored (rejected for analysis). Default: \( \max(c(2, \text{floor}(\log(b(length(x), \text{base}=2))))). \)

n.taper an integer specifying the number of sinusoidal tapers to use in developing the eigenspectra for each block of the time series. Default: 5.

recenter a logical value. If TRUE, the mean value of the data is subtracted from the tapered series prior to forming a multtaper SDF estimate of the input time series. Default: TRUE.

significance the significance is the number of times you expect the underlying hypothesis of stationarity to fail even though stationarity remains true. Essentially, you are allowing for error in the result. A significance of 0.05 means that you are allowing 5 percent error, i.e., you are 95 percent confident in the result. Default: 0.05.

Details

The algorithms is outlined as follows:

1. Re-centering The time series \( x \) is recentered by subtracting the sample mean.

2. Blocking The recentered series is then segmented into \( n.block \) non-overlapping blocks in time.

3. Mutitaper SDF estimation For each block, \( n.taper \) eigenspectra are formed by calculating the periodogram of the block windowed by each of the \( n.taper \) tapers. These eigenspectra are then averaged to form a multitaper SDF estimator for the current block.

4. ANOVA table A subset of each multitaper SDF estimate is formed by extracting only those values corresponding to frequencies which are approximately uncorrelated (the details of this exercise can be found in the references). Each subset (one per block) is stacked in rows such that an \( n.block \times M \) matrix (\( S \)) is formed, where \( M \) is the number of (subset) Fourier frequencies. The (two-factor) ANOVA table (\( Y \)) is then formed via \( Y \equiv \log(S) - \psi(n.taper) + \log(n.taper) \), where \( \psi(\cdot) \) is the digamma function and \( \log \) is the natural logarithm function.

5. PSR statistics Using the ANOVA table and (row, column, and grand) means of the ANOVA table, the Priestley-Subba Rao statistics are generated: one for investigating time effects, one for investigating frequency effects, and one which combines the two to test time-frequency effects. See references for details.

6. Stationarity tests The PSR statistics are then compared to corresponding chi-square \( ((1 - \text{significance}) \times 100) \) percentiles (normalized by \( \psi'(n.taper) \) where \( \psi'(\cdot) \) is the trigamma function). Specifically, if the PSR statistic is found to be greater than the corresponding chi-square percentile, it indicates that there is a \( 1 - \text{significance} \) probability that the data is nonstationary.

Value

an object of class \texttt{stationarity}. 
S3 METHODS

\texttt{as.list} convert output to a list.
\texttt{print} prints the object. Available options are:
\begin{itemize}
  \item \texttt{justify} text justification \texttt{ala prettPrintList}. Default: \texttt{"left"}.
  \item \texttt{sep} header separator \texttt{ala prettPrintList}. Default: \texttt{":"}.
  \item \texttt{n.digits} number of digits \texttt{ala prettPrintList}. Default: \texttt{5}.
\end{itemize}
\texttt{...} Additional print arguments sent directly to the \texttt{prettyPrintList function}.
\texttt{summary} prints a summary of the stationarity test results.

References


See Also

\texttt{hurstBlock}.

Examples

```r
## assess the stationarity of the ecgrr series
z <- stationarity(ecgrr, n.block=8)
## print the result, noting that all tests fail.
## The strongest failure attributed to the
time-based fluctuations of the eigenspectra
print(z)

## print a summary of the results, including the
## ANOVA table of the eigenspectra
summary(z)
```

\begin{center}
\begin{tabular}{ll}
\textit{surrogate} & \textit{Surrogate data generation} \\
\hline
\end{tabular}
\end{center}

Description

This function can be used to generate surrogate time series via various frequency domain bootstrapping techniques. Bootstrapping has been used (in the statistics community) to assess the sampling variability of certain statistics. The nonlinear dynamics community typically uses bootstrapping to detect nonlinear structure in stationary time series. Given a time series, this function is used to generate surrogate series via Theiler’s Amplitude Adjusted Fourier Transform (AAFT), Theiler’s phase randomization, Davies and Harte’s Circulant Embedding (CE) technique, or Davison and Hinkley’s (DH) phase and amplitude randomization technique.

Theiler’s techniques produce so-called \textit{constrained realizations} since some statistical aspect of the original data preserved (the histogram for the AAFT and the periodogram for the phase randomization). The other techniques, circulant embedding and Davison-Hinkley, are non-constrained as both the amplitudes and phases of the original series are randomized.
Usage

surrogate(x, method="ce", sdf=NULL, seed=0)

Arguments

x         a vector containing a uniformly-sampled real-valued time series.
method    a character string representing the method to be used to generate surrogate data.
          Choices are:
          "aaft"  Theiler's Amplitude Adjusted Fourier Transform.
          "phase" Theiler's phase randomization.
          "ce"    Davies and Harte's Circulant Embedding.
          "dh"    Davison and Hinkley's phase and amplitude randomization.
          Default: "ce".

sdf       an object of class SDF, containing a single-sided spectral density function estimation (corresponding to the original data) over normalized frequencies \( f_k = k/(2N) \) for \( k = 0, \ldots, N \) where \( N \) is the number of samples in the original time series. This argument is only used for the circulant embedding method. Default: NULL unless the circulant embedding method is used, and then it is SDF(x, method="multitaper", recenter=TRUE, taper=h, single.sided=T) where \( h = \text{taper(type="sine", n.sample=N, n.taper=5, norm=TRUE)} \).

seed      a positive integer representing the initial seed value to use for the random number generator. If seed=0, the current time is used as a means of generating a (unique) seed value. Otherwise, the specified seed value is used. Default: 0.

Details

The algorithms are detailed as follows:

phase  The discrete Fourier transform of a time series is calculated and the phase at each frequency is randomized to be uniformly distributed on \([0, 2\pi]\). Phase symmetry is preserved so that an inverse DFT forms a purely real surrogate. Null hypothesis: the original data come from a linear Gaussian process. Side effect: the periodogram of the surrogate and original time series are the same.

aaft   An \( N \)-point normally distributed realization of a white noise process is created, where \( N \) is the length of \( x \), and sorted to have the same rank as \( x \) (e.g., if rank\((x_t) = 5 \) it means that \( x_t \) is the fifth smallest element of \( x \)). The result is then phase randomized and its rank \((r)\) is then calculated. The surrogate is then created by rank ordering \( x \) using \( r \). Null hypothesis: the observed time series is a monotonic nonlinear transformation of a Gaussian process. Side effect: the amplitude distribution (histogram) of the surrogate and original time series are the same.

ce     The circulant embedding technique is based upon generating surrogates whose estimated SDF (e.g., a periodogram) is not constrained to be the same as that of the original series (for references for details).

dh     The Davison-Hinkley technique is based upon generating surrogates by randomizing both the phases and the amplitudes in the frequency domain followed by an inversion back to the time domain.
**Value**

an object of class `surrogate`.

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`plot` plots the surrogate data realizations. The following options may be used to adjust the plot components:

- **show.** A character string defining the data to display. Choices are "series", "surrogate", or "both" for plots corresponding to the original series, surrogate series, or both, respectively. Default: "surrogate".
- **type** Character string denoting the type of data to plot. Options are "time" for time history, "sdf" for a multitaper spectral density function estimation, "pdf" for a probability density function estimation, and "lag" for a two-dimensional embedding (lag plot). Default: "time".
- **stack** A logical flag. If TRUE, the `stackPlot` function is called as opposed to the default plot function. As `stackPlot` requires a common abscissa, this option is only available for `type="time"` (time history) or `type="sdf"` (spectral density function plot). Default: TRUE.
- **xlab** Character string denoting the x-axis label for the "time" and "sdf" "pdf" types. Default: "Time", the series name, and "Frequency (Hz)", respectively.
- **ylab** Character string denoting the y-axis label for the "time" style. Default: the series name.
- **cex** Character expansion factor (same as the `cex` argument of the `par` function). Default: 1.
- **adj.main** Title adjustment ala the `adj` argument of the `par` function). Default: 1.
- **line.main** Line spacing for title ala the `line` argument of the `text` function). Default: 0.5.
- **col.series** A character string or integer denoting the color to use when plotting data corresponding to the original series. See the `colors` function for more details. Default: "black".
- **col.surrogate** A character string or integer denoting the color to use when plotting data corresponding to the surrogate series. See the `colors` function for more details. Default: "red".
- **...** Additional plot arguments (set internally by the `par` function).

`print` prints a summary of the surrogate data realization. Available options are:

- **...** Additional print arguments used by the standard `print` function.

**References**


See Also

`infoDim`, `corrDim`.

Examples

```r
## create surrogate data sets using circulant
## embedding method
surr <- surrogate(beamchaos, method="ce")

## print the result
print(surr)

## plot and compare various statistics of the
## surrogate and original time series
plot(surr, type="time")
plot(surr, type="sdf")
plot(surr, type="lag")
plot(surr, type="pdf")

## create comparison time history
plot(surr, show="both", type="time")
```

timeLag  

*Estimate the proper time lag for single variable delay embeddings*

Description

Given the time series $X_t$, the embedding dimension $E$, and the time lag $\tau$, the embedding coordinates are defined as $X_t, X_{t-\tau}, \ldots, X_{t-(E-1)\tau}$. This function can be used to estimate the time lag $\tau$ using a variety of statistical methods.

Usage

```r
timeLag(x, method="acfzero", plot.data=FALSE)
```
Arguments

- **x**: a vector containing a uniformly-sampled real-valued time series.
- **method**: character string denoting the method to use in estimating the time delay. Supported methods are:
  - "acfzero": First zero crossing of the autocorrelation function.
  - "acfdecor": First 1/exp of the autocorrelation function.
  - "acfnadir": First nadir of the autocorrelation function.
  - "mutual": First nadir of the average mutual information function.
  Default: "acfzero".
- **plot.data**: a logical value. If TRUE, a plot of the time lag selection process is displayed. Default: FALSE.

Details

Currently, there exists no single method which yields an optimal time lag estimation although there are some basic criteria that are used: if the lag is chosen too small, the coordinates will be too highly correlated and the embedding will cluster tightly around the diagonal in the phase space. If the lag is chosen too large, the resulting coordinates may be almost uncorrelated and the resulting embedding may become unduly complicated, even if the underlying attractor is not. The goal is to find a lag which falls in-between these scenarios.

In addition the autocorrelation-based methods this function supports an estimation method based on the time-delayed mutual information (TDMI), which can be thought of as a nonlinear generalization of the autocorrelation function. For a random process \( X_t \) the TDMI, \( I(\tau) \), is a measure of the information about \( X_{t+\tau} \) contained in \( X_t \). The first nadir of \( I(\tau) \) gives the delay, \( \tau_0 \), such that \( X_{t+\tau_0} \) adds maximal information to that already known from \( X_t \). This \( \tau_0 \) is returned as an estimate of the proper time lag for a delay embedding of the given time series.

Value

an integer representing the estimated time lag.

References


See Also

embedSeries, infoDim, corrDim, lyapunov, findNeighbors, KDE, determinism.
Examples

```r
## estimate the proper time lag for an embedding
## of the beamchaos data using the first zero
crossing of the ACF
as.numeric(timeLag(beamchaos, method="acfzero", plot=TRUE))
```
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